MATH 221, Spring 2010.

Name _____

ID _____

1. Solve the following system of equations.

$$x + 2y + z = 1$$
$$2x + 5y + z = 3$$
$$y - z = 1$$

2. Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$. Find all matrices of the form $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $AM = MA$.

3. Suppose A and B are 3×3 invertible matrices where $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 5 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ and

$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & 3 \end{bmatrix}$.

(a) Solve the system of equations $(A^{-1}B^{-1})X = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

(b) Solve the system of equations $(A^{-1})^T X = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

4. If A is a 2×2 matrix such that $A \begin{bmatrix} 3 & 2 \\ 4 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = A \begin{bmatrix} 2 & 1 \\ 4 & -2 \end{bmatrix}$ find A.

5. (a) Find the determinant of $\begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 2 & 1 \\ 0 & 3 & 1 & 2 \\ 2 & 4 & 9 & 1 \end{bmatrix}$.

(b) Find all a so that $\begin{bmatrix} 1 & 2 & a \\ 0 & 2 & 1 \\ 4a & 0 & 1 \end{bmatrix}$ is invertible.

6. Let A, B and C be 3×3 matrices. If $det(A) = 2.det(B) = 3$ and $det(C) = 634652645/12364737$ find the following

(a) $det(2A^2B^{-1})$

(b) $det(C^{-1}BC)$

7.

If $A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}$ and $det(A) = 2010$ find x_1 and x_2 where $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} a + 2b \\ e + 2f \\ i + 2j \\ m + 2n \end{bmatrix}$.